The Use of a Geographic Information System for Second-Order Analysis of Spatial Point Patterns

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Abstract

The application of second-order neighborhood analysis for determining clustering tendency in spatial point patterns is examined within a GIS context. A computer program implementing the second-order methods discussed and demonstrated by Getis (1983; 1984; 1988; 1990) has been developed for the GRASS GIS. A variety of sample point patterns are analyzed using these methods to illustrate the advantages of incorporating spatial statistics such as these within a GIS.

Introduction

Geographical phenomena often occur in the form of point distributions and the analysis of such data involves the identification, description and modeling of the resulting spatial patterns. A large body of techniques for this purpose have been developed over the years, most notably various autocorrelation statistics (Cliff and Ord, 1973). One particular interest is in determining whether the data exhibits a predisposition to cluster, that is, to form natural groupings. This task is actually more difficult to perform than one might think. Although there are many algorithms for creating clusters from a data set, the literature on clustering tendency is very limited and the problem has actually been virtually ignored in many applications (Dubes and Jain, 1976, 1979). There are several reasons for this situation including the fact that there exists few suitable theories as to how to define "clustering structures" and just as few practical procedures for identifying this tendency. In addition, clustering tendencies are also scale dependent and the observed clusters of points may be real or only apparent because of the size of the region that was chosen or observation (Hsu and Tiedemann, 1968). Determining an appropriate sampling area can also be difficult as variations (nonuniformity) in the underlying distribution of points can affect tests of clustering tendency.

Discriminating between random, clustered and regular patterns of data is a fundamental problem in exploratory data analysis (Panayirci and Dubes, 1987). Within this context a number of techniques are being brought together to address this problem. A potential addition to these techniques is an approach known as second-order neighborhood analysis. Based on second-order methods introduced by Ripley (1976; 1977), the approach has been discussed and extensively demonstrated by Getis (1983; 1984; 1988). Early applications of second-order analysis examined point patterns and tested random hypotheses, often based on the Poisson distribution. Getis (1990) has extended the approach by recently introducing a family of measures called G-statistics.

This paper examines the application of second-order neighborhood analysis, in particular the L-statistic and G-statistic, for determining clustering tendency in point patterns. Just as exploratory data analysis has been greatly facilitated by advances in computing which permit large amounts of data to be easily manipulated, the spatial analysis of geographical data can benefit from the availability of geographical information systems. Few such systems currently include the capability for performing spatial statistics. An additional objective of this paper is to show how techniques for point pattern analysis can easily be incorporated into an existing GIS.

Implementation

In order to evaluate the use of second-order methods for analysing spatial point patterns, a computer program Gstat implementing these statistics was written for the Geographical Resources Analysis Support System (GRASS) version 3.1. GRASS is a fairly sophisticated geographical information system developed by the U.S. Army Corps of Engineers (Westervelt et al., 1986). Although originally designed to support environmental resource management for the military, GRASS has since evolved into a large general-purpose GIS that is currently being used for a wide variety of applications in many different disciplines. There are several reasons why GRASS is particularly well suited for this particular application. It’s availability on computer workstations such as SUN computers means that GRASS is quite versatile, such as the capability of having multiple "windows" for displaying both graphics and commands at the same time (Figure 1). In exploratory data analysis it is useful to be able to see the data while also computing various statistics. The fact that GRASS is in the public domain also facilitates adding new programs to its already extensive collection. As illustrated by the manual page which follows this text, the Gstat program was developed to look like any other GRASS command.

While primarily grid-based, GRASS also supports vector and point data. The latter are stored in "site" files and it is these data that are used by the Gstat program. A site file consists of a series of coordinates, one per line, consisting of two or three fields (Figure 2). The first two fields are the easting and northing values and the optional third field is a numeric attribute, usually an integer value. One limitation of the version of GRASS used here (version 3. 1) is that only UTM coordinate data are supported. The numeric

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attribute can represent anything including weighted point values as used for the G-statistic. In the example of Figure 2, all of the points happen to have an attribute of "99". Site files can easily be created, edited and accessed using GRASS. Site files containing sample data were created for use in this investigation.

As described in the manual page for the Gstat program (Appendix 1), a number of second-order methods have been implemented. These include the L(d) statistic for point pattern analysis (Getis, 1984; Getis and Franklin, 1987) and G(d) statistics for weighted point associations (Getis, 1990). Statitistics for both individual points and the overall study area can be computed. The equations for the L(d) and G(d) statistics are described in Appendix 2.

Any distance d can be specified (sample output is show in Figure 16 and 17). The Gstat program is easily invoked on the command line from within GRASS. The first example below shows how to obtain the G-statistic for the points in the site file named “sample1" using a distance of 10 meters. The second example shows the L-statistics computed for the same distance. The 5% and 1% acceptance for statistical significance are estimates from Ripley (1978; 1979) as also used by Getis and Franklin (1987):

```
GRASS-GRDD> Gstat -f 10 sarnple1

G(d) Statistic for window  600050(E) 600100(E) 4900050(N) 4900100(N)

number of points: 140       area: 2500.000 sq. meters Case: i <> j

distance  G(d)      E(d)      V(d)      z-score

10.000    0.471     0.143     0.031     1.868

GRASS-GRID> Gstat -fb 10 sample1

G(d) Statistic for window 600050(E) 600100(E) 4900050(N) 4900100(N)

number of points: 140       area: 2500.000 sq. meters  Case: i <> j

distance            L                   5% acceptance                 1% acceptance

10.000              4.800               4.489     5.511               -2.125    12.125
```

Output from this program can also be redirected to a file and used as input to a graph plotting program as was used to generate the various graphs show in the figures. What is convenient about using a GIS on a workstation is that all of these commands can easily be executed and the resulting output shown on the computer screen at the same time.

It is fairly intuitive that the analysis of point patterns is not only dependent upon the size and shape of the study area but is also affected by the presence of boundaries. Several approaches have been suggested to handle this edge effect (Ripley, 1981) although there appears to be no standard solution and the selection of a particular approach is highly dependent upon the nature of the data itself. It may be appropriate to compare the results using different approaches to determine how much difference exists between them. One approach is to consider a study area embedded in a larger domain so that measurements can be made to points outside &.e study area. This method is usually referred to as that of allowing a "border" or "guard area". Although this approach is convenient, it requires a larger number of data points to be available and also restricts the potential study area. For this reason it is often difficult to use with real data sets. It is, however, the approach used in this investigation as theoretical distributions of data points could be generated for any size of area. In GRASS it is possible to specify a rectangular sampling "window" defining the study area from which data are to be selected. If the study area is rectangular and it can be assumed that a similar pattern of points would hold in a larger context then the region can be regarded as a torus, so that points on opposite edges are considered to be close. Directional tendencies observed in a point pattern would suggest that an alternative approach be used. Toroidal edge correction has been included as a option in the Gstat program.

Analysis

As implemented, the Gstat program is quite versatile and any number of point patterns at any scale can be examined using one of the second-order methods. Although not pursed in this study, it is also quite easy to select various subareas for the point data so that the sensitivity of the statistics to the location of study areas can be examined. Five different point patterns are presented here. They are standard patterns to be used to investigate the use of second-order neighborhood methods. The first four of the point patterns involve unweighted points while the fifth pattern is a simple example of weighted points. The potential complexity of varying both spatial patterns and point weights makes it difficult to select appropriate sample data that would clearly evaluate the properties of these statistics. Certainly many other variations of point data could be examined. In fact, another potential application is to compare various theoretical point distributions against a real data set. The examples used here illustrate how that might be done.

The four unweighted point patterns include a random point distribution (Figure 3), a square grid (Figure 4), a hexagonal grid (Figure 5), and a regular pattern of clustered points (Figure 6). All of these point patterns are within a 50 x 50 meter window with a 1-meter sampling interval. The random point distribution has 5% of the potential point locations occupied by data points. The points in both the grid patterns are spaced at 5 meter intervals in both the east-west and north-south directions. For the clustered point pattern, the center of each cluster is separated by 10 meters from adjacent clusters and within each cluster the points are separated by 2 meters.

For each of these four patterns both L(d) and G(d) statistics were computed and plotted with the distance d being incremented by 2 meter intervals (Figures 7-14). Statistical significance could only be ascertained for the L(d) statistic as the expected values of G(d) cannot be computed for unweighted points (it is the same value as G(d) itself). In this case, an approximation of the 5% significance points as derived by Ripley (1979) were used. These values are $\pm 1.45 \sqrt{A}/(n-1)$ where A is the area of the study area. These limits have been plotted on graphs of L(d) as dashed lines.

The plots of L(d) and G(d) for the random point distribution (Figure 7 and Figure 8) are what one would expect and they serve as a basis of comparison for the other three nonrandom point patterns. None of the L(d) values are significantly different from a pattern produced by a Poisson process in the plane although there is a tendency for the observed values to be slightly lower than the expected L(d) values. G(d) for this even concentration of points exhibits a steadily increasing curvilinear graph since G(d) is a cumulative measure.

The two grid patterns represent an interesting comparison with the random point pattern and between themselves. The points in the hexagonal grid (Figure 9) are more closely packed than those in the square grid (Figure 11) and this difference is shown in the plots of L(d) for each. Most of the L(d) values for the hexagonal grid are significantly different from a Poisson process. The low values of L(d) indicate dispersion or inhibition rather than clustering. A larger distance d is required for the square grid before all of the L(d) are significantly different from a Poisson process. Both of the G(d) graphs for these two grid patterns are similarly irregular with a stepped pattern although the irregularity of the square grid (Figure 10) is much more pronounced than that of the hexagonal one (Figure 12).

The regular clustered pattern of points also shows some distinctive trends in its L(d) and G(d) graphs (Figure 13 and Figure 14). At distances of 2 and 4 meters the L(d) values are significantly higher than expected from a Poission process and this is indicative of clustering. This pattern is again repeated at 12 and 14 meters which represents the interval spacing between the point clusters. The undulating shape of the G(d) graph is indicative of a non-random pattern of points but is much harder to interpret than the L(d) graph.

Finally, a simple pattern of weighted points was examined to obtain some sense of how the G-statistic characterizes such data. The spatial pattern was kept simple (only 15 points) so that the effect of weighting could be more clearly discerned. The point values along with the spatial location of the high (H) and low (L) points are indicated in Figure 15. A 50 x 50 meter window was also used for the sample area and toroidal edge correction was applied. The point values along with the G(d) and Gi(d) statistics for d = 5 meters are shown in Figure 16 and the G(d) and Gi(d) statistics for d = 15 meters in Figure 17. At neither of these two distances nor any other distance was the computed overall z-score significant at the 5% level. However, an examination of the individual z-scores for each point reveals that several of the higher weighted points are significant and indicate clustering of the high values. The z-score for point number 3 (600068E, 490079N) over a range of distances d is shown in Figure 18. It indicates clustering of high values at this point up to a range of 12 meters. Although contrived, this sample data illustrates a weakness of relying only upon a single overall statistic. Unlike autocorrelation statistics such as Moran's I, it is advantageous that Gi(d) values can be examined to determine local associations.

Conclusions

This investigation has just touched upon some of the properties of second-order methods for investigating clustering of point patterns. It is clear from the sample data that the L(d) and G(d) statistics can provide additional useful information about the spatial concentration of point data although its sensitivity to various conditions such as study area size and boundary effects still needs to be determined. The use of second-order methods is also not without problems (Baddeley and Silverman, 1984) and therefore should be compared with other point pattern statistics. Incorporating such spatial statistics into a GIS greatly facilitates their use and easily permits many further investigations of this type.

Acknowledgements

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APPENDIX 1 : GRASS Manual Page for the *Gstat* Program

NAME

Gstat – Compute the G-statistic for a named site file
(G Language Tool)

SYNOPSIS

Gstat [-abcef] *distance sitefile*

DESCRIPTION

Gstat computes a variety of measures of spatial association (clustering) for point pattern analysis using the data points contained in a GRASS sites file. These measures, known as G–statistics, are based on second-order analysis. They determine the proportions of (weighted) points within a specified distance of individual points relative to all (weighted) points in the study area. Both simple points patterns and weighted point associations can be studied. The statistic for simple point patterns L(d) is based on comparisions with a Poisson distribution. Approximations of the 5% and 1% significance points are also computed for significance testing of L(d). The G(d) statistic measures the concentration or lack of concentration of the weighted point locations in the study area. Expected values and a z-score are also computed for hypothesis testing. G-statistics can be computed for both individual points (Li, Gi) and overall for the study area (L, G).
The study area is determined either by a pre-defined GRASS "window" or the total number of points in a site file.

OPTIONS

-a This option causes the values of a point to be included with those within a radius of itself. That is, any clustering includes the point itself.

-b This option specifies that only the position of the points will be included in computing a clustering statistic, not the value of the points. In effect, an L(d) statistic rather than a G(d) statistic is computed.

-c This option causes the Li(d) or Gi(d) statistic for each point to be printed in addition to the overall statistics for the study area.

-e This option specifies that the study area should be treated as a torus to handle edge effects.

-f This option produces a full annotated output rather than only printing the statistics themselves. Without this option the output can readily be redirected to a file for further analysis or plotting.

NOTES

Note that output from the GRASS *Gstat* command can be redirected into a file for further manipulation or display.

SEE ALSO

Dpoints[2D] To create and display points in the current window
Gsites[2G] Lists the point coordinates in a site file
window[1] Window management system
d.sites[1] Displays point sites

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APPENDIX 2 : Equations for Second-Order Statistics, L(d) and G(d)

The equations for the L(d) and G(d) statistics are shown below. Statistics can be computed for both individual points (L_i and G_i) and for the entire study area (L and G) at different distances d. The derivations of L(d) and G(d) and related statistics are further described in Getis (1984) and Getis (1990).

$$L_i(d) = \left[A \sum_j w_{ij}(d) / \pi (n - 1) \right]^{1/2} \qquad L(d) = \sum_i L_i(d) / n$$

d = radius of interaction

A = area of rectangular study region

Σ w(d) = summation of all points j within distance d of point i ($j \neq i$)

n = total number of points within study region

(n - 1) = number of possible pairs of points including point i in each pair

$$G_i(d) = \frac{\sum_j w_{ij}(d)\ x_j}{\sum_j x_j} \qquad G(d) = \frac{\sum_i \sum_j w_{ij}(d)\ x_j}{\sum_i \sum_j x_j}$$

x = weight at some point i

Σ w(d) x = summation of the weights for all points within distance d of a point i ($i \neq j$)

Σ x = summation of the weights for all points

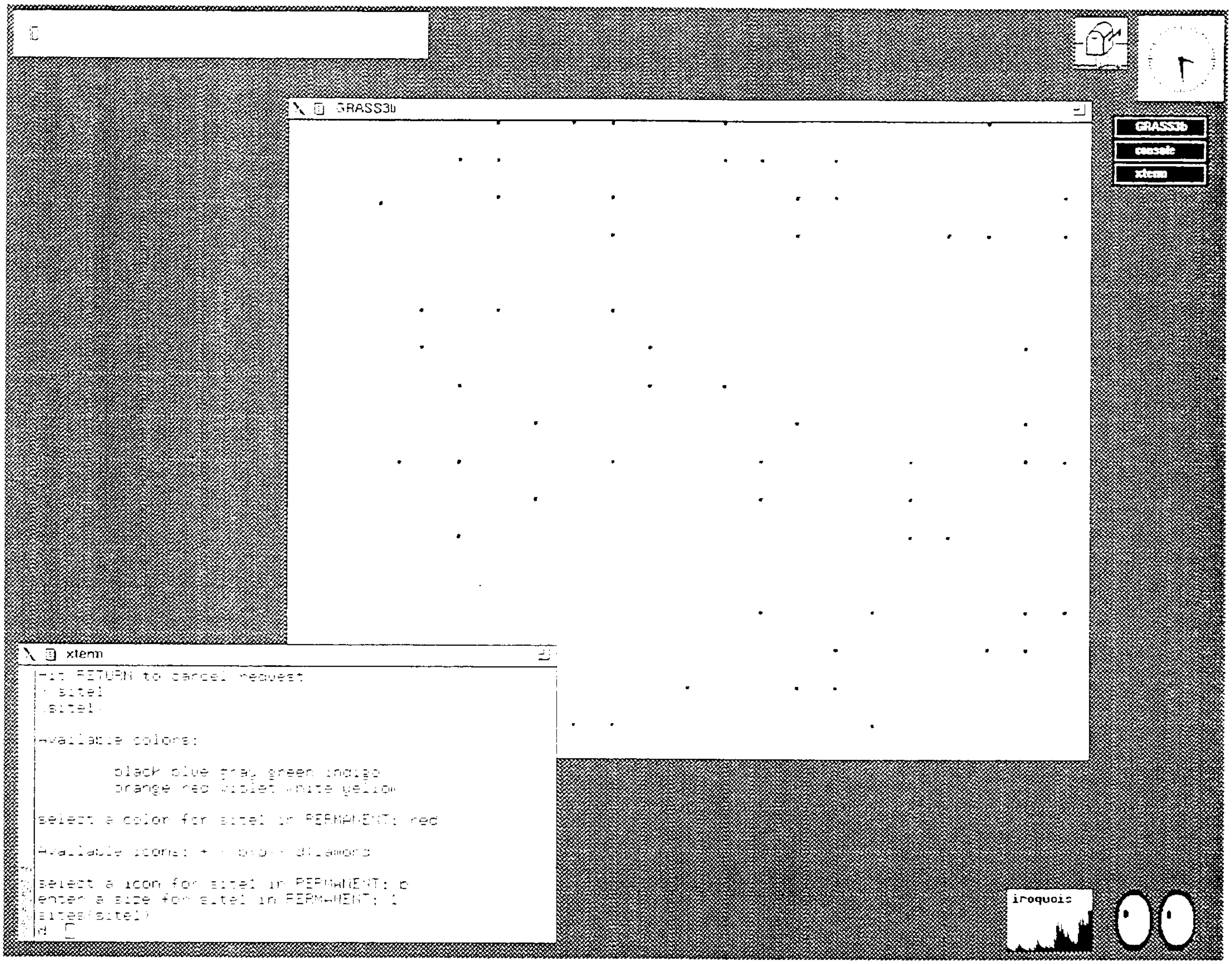
GRASS3b
console
xterm
iroquois
to cancel
Available colors:
black blue gray green indigo
orange red violet white yellow
select a color for site1 in PERMANENT: red
select a icon for site1 in PERMANENT:
enter a size for site1 in PERMANENT:

Figure 1

```
name|site7
desc|Random sites

# <easting>|<northing>|<description of the site>
#
# (note: the above is the format of a point line)
# (lines that begin with # are comments.        )
# (actual point lines should not have the #     )

600062|4900070|#99
600061|4900070|#99
600062|4900072|#99
600061|4900071|#99
600060|4900070|#99
600060|4900068|#99
600061|4900069|#99
600062|4900068|#99
600064|4900068|#99
600062|4900067|#99
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600059|4900087|#99
600056|4900086|#99
600084|4900072|#99
600084|4900074|#99
600082|4900078|#99
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600082|4900076|#99
```

Figure 2

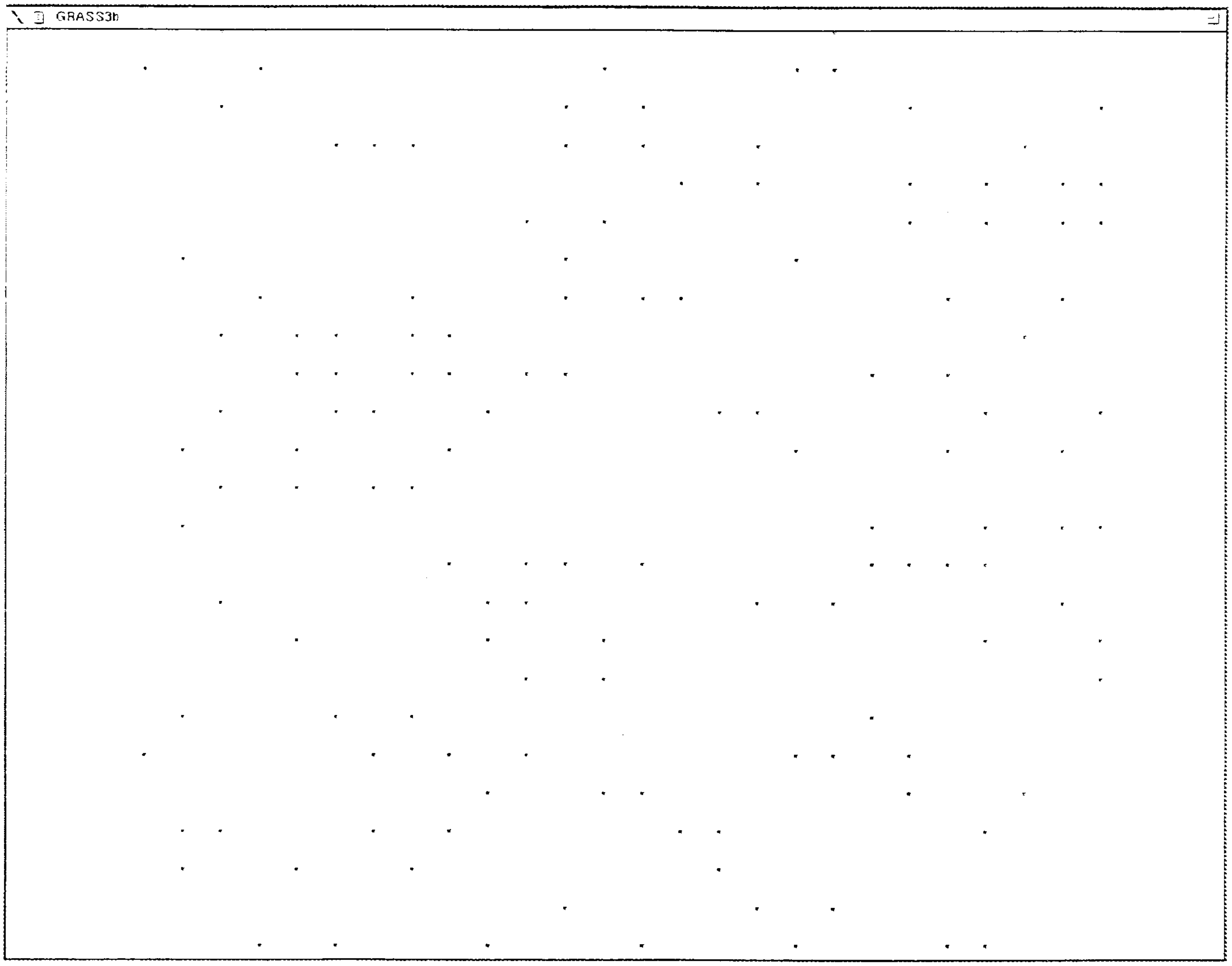


Figure 3

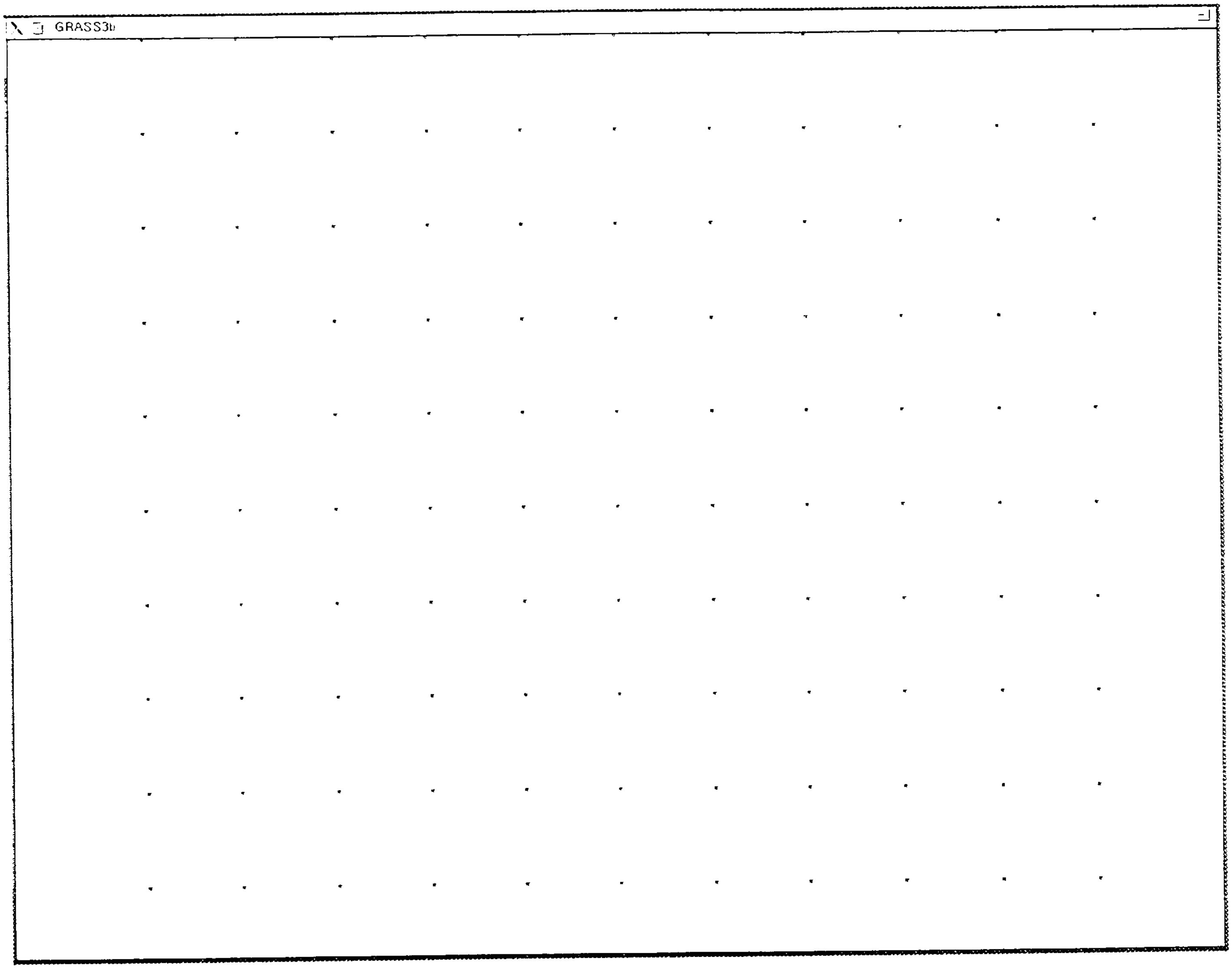


Figure 4

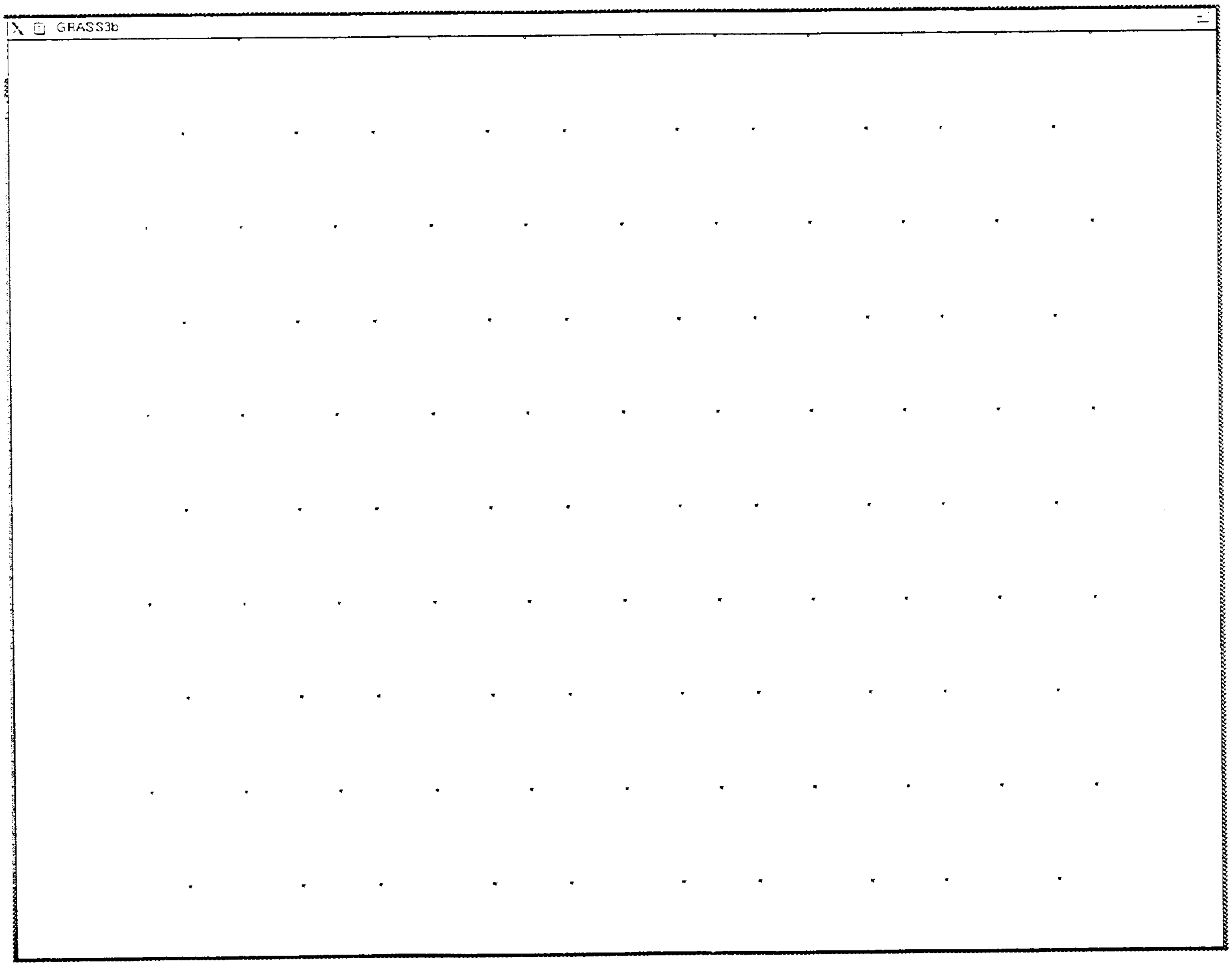


Figure 5

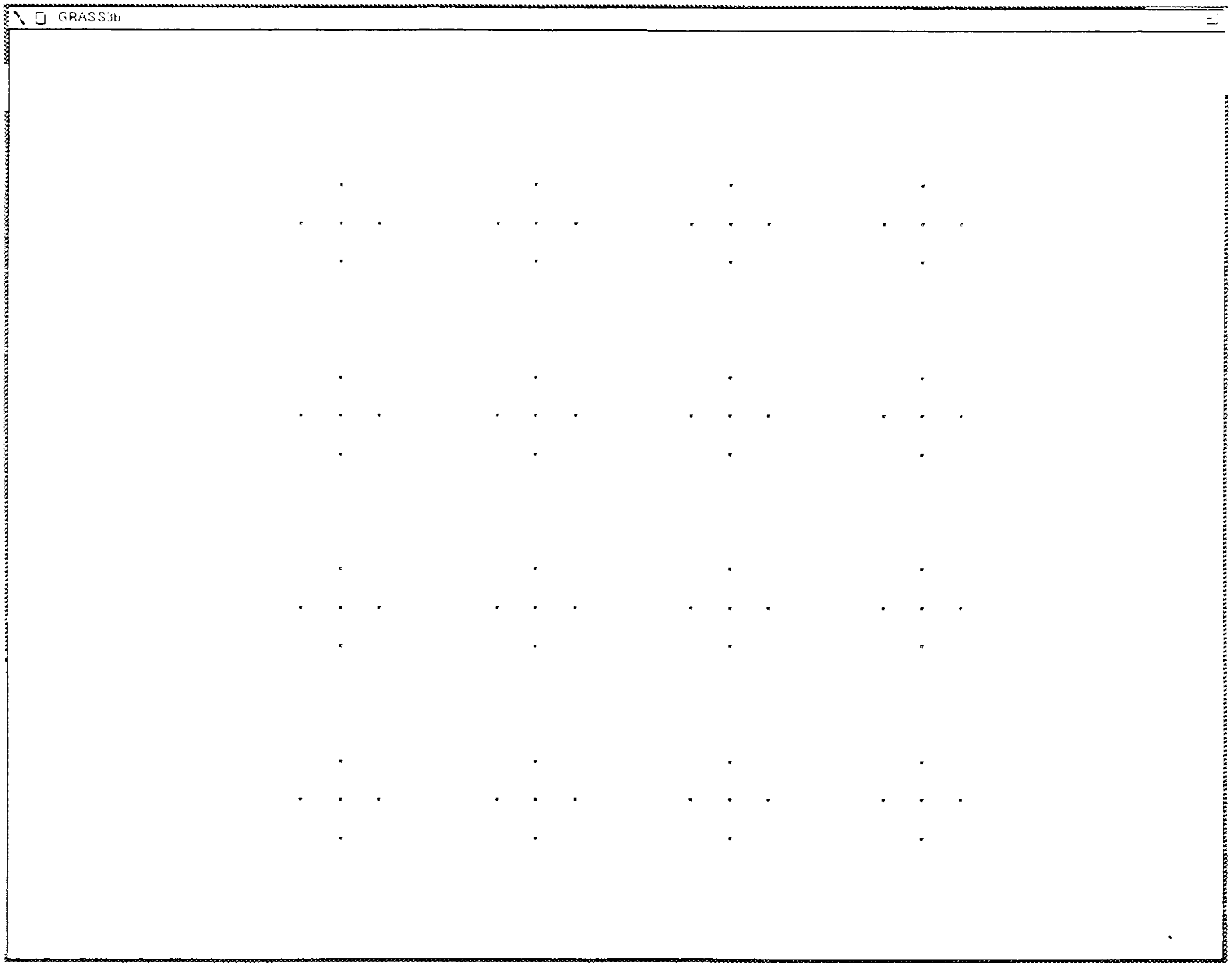


Figure 6

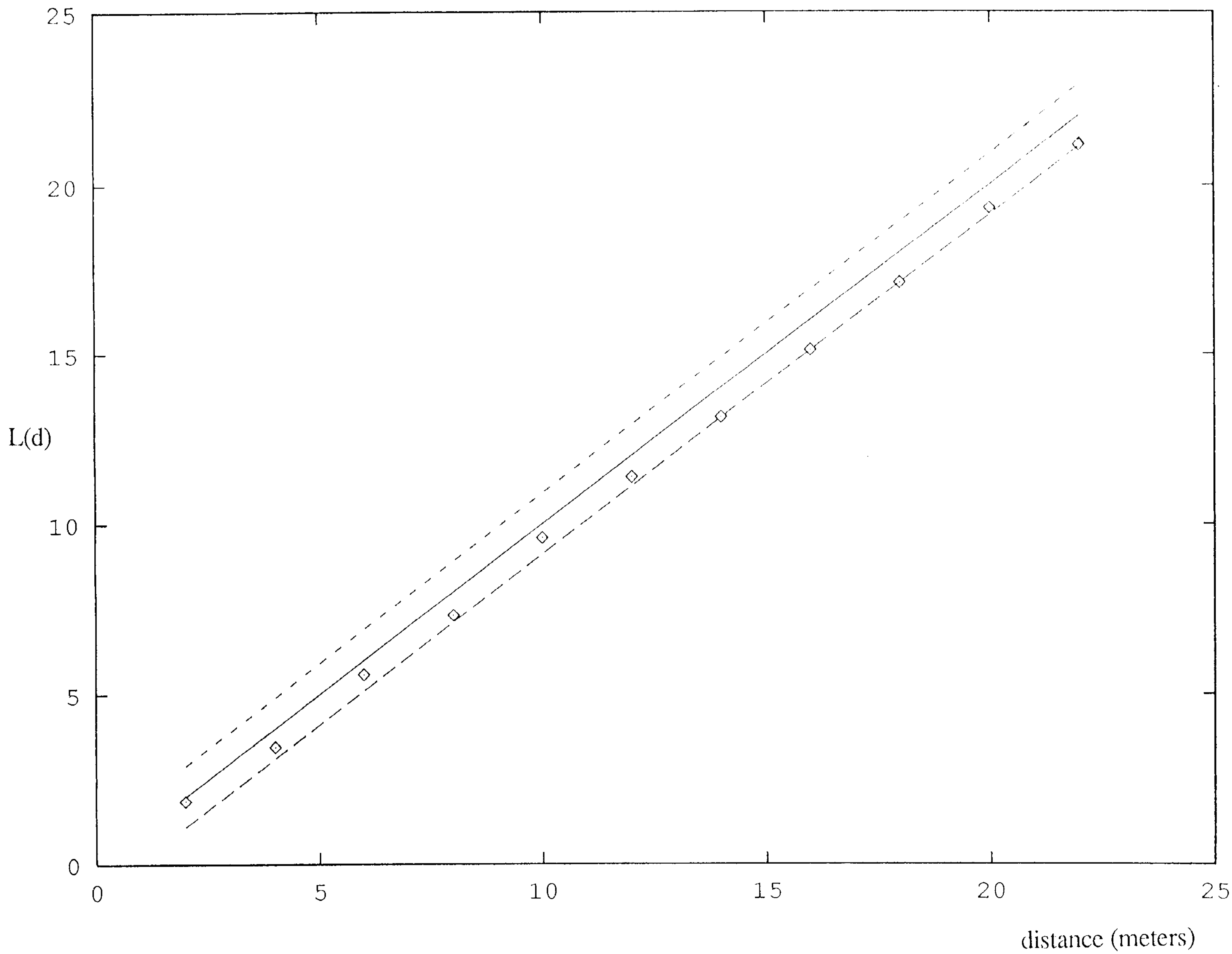


Figure 7

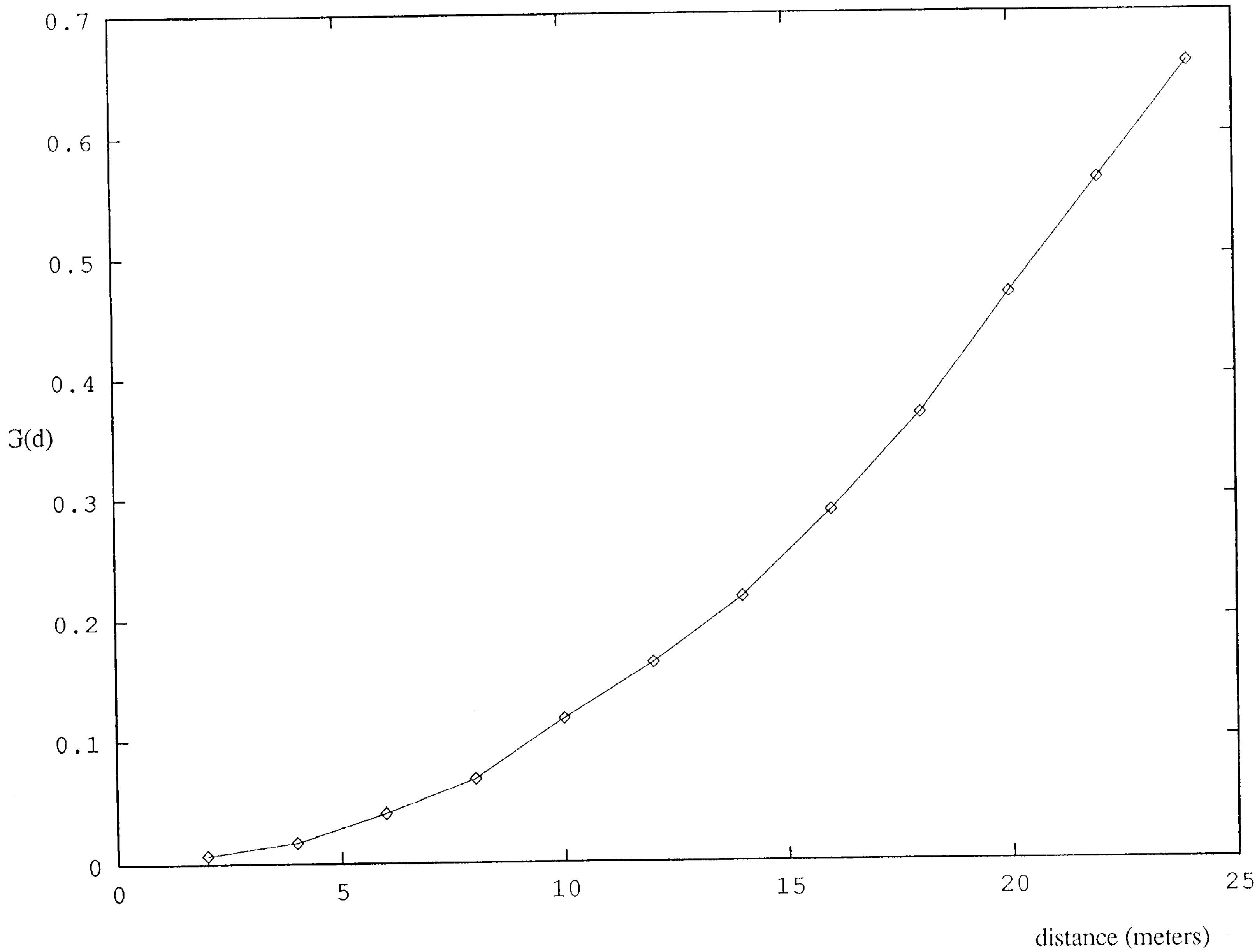


Figure 8

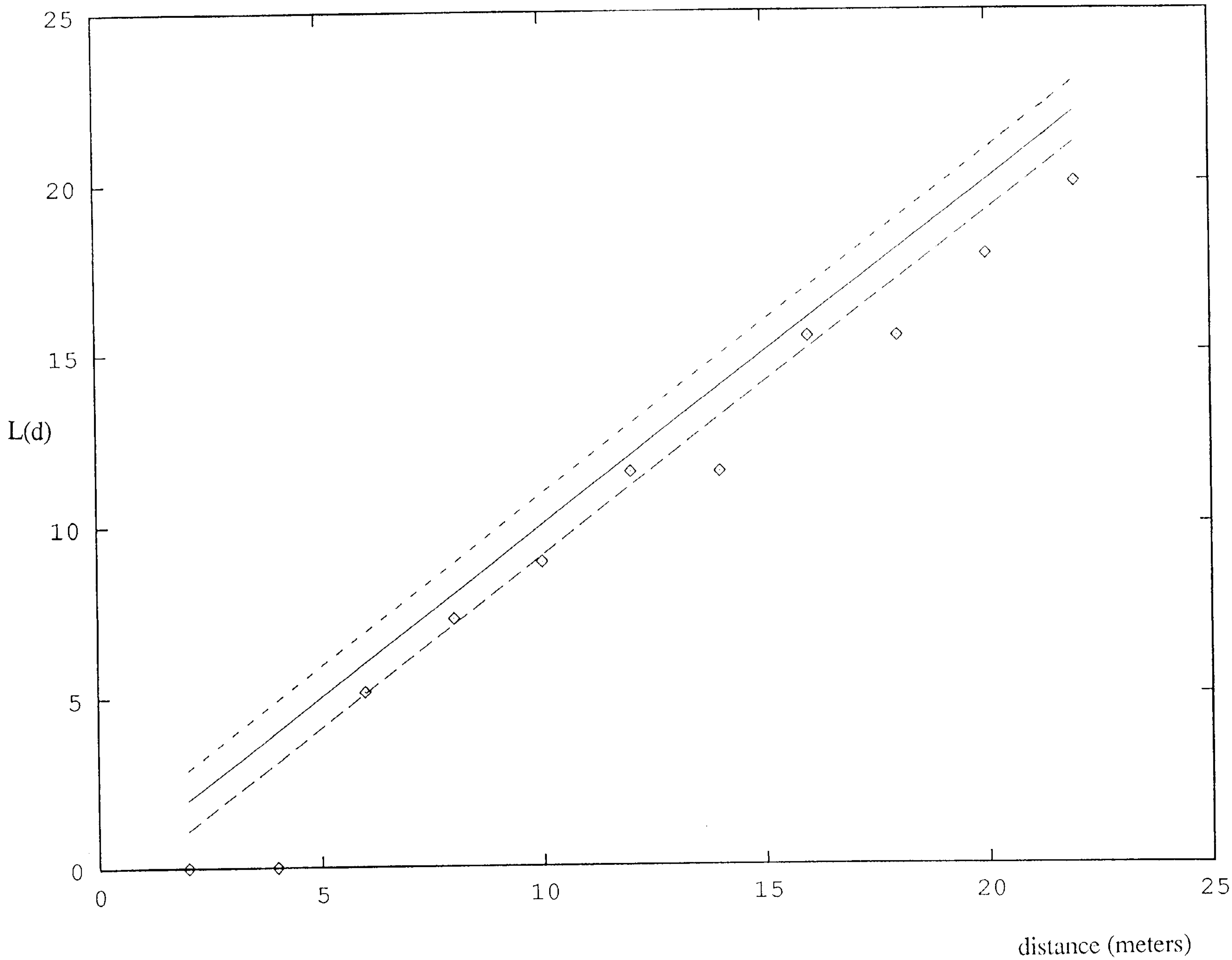


Figure 9

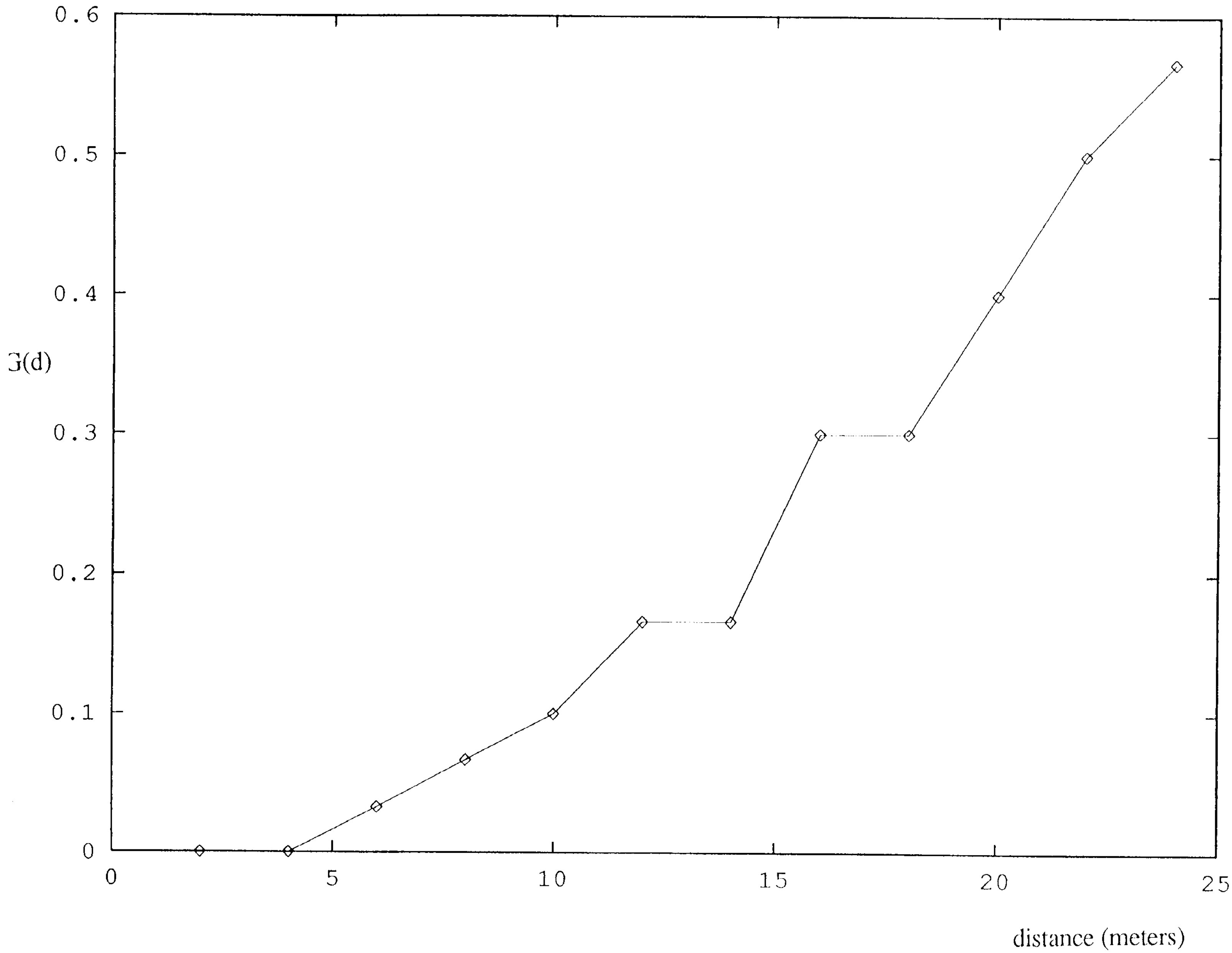


Figure 10

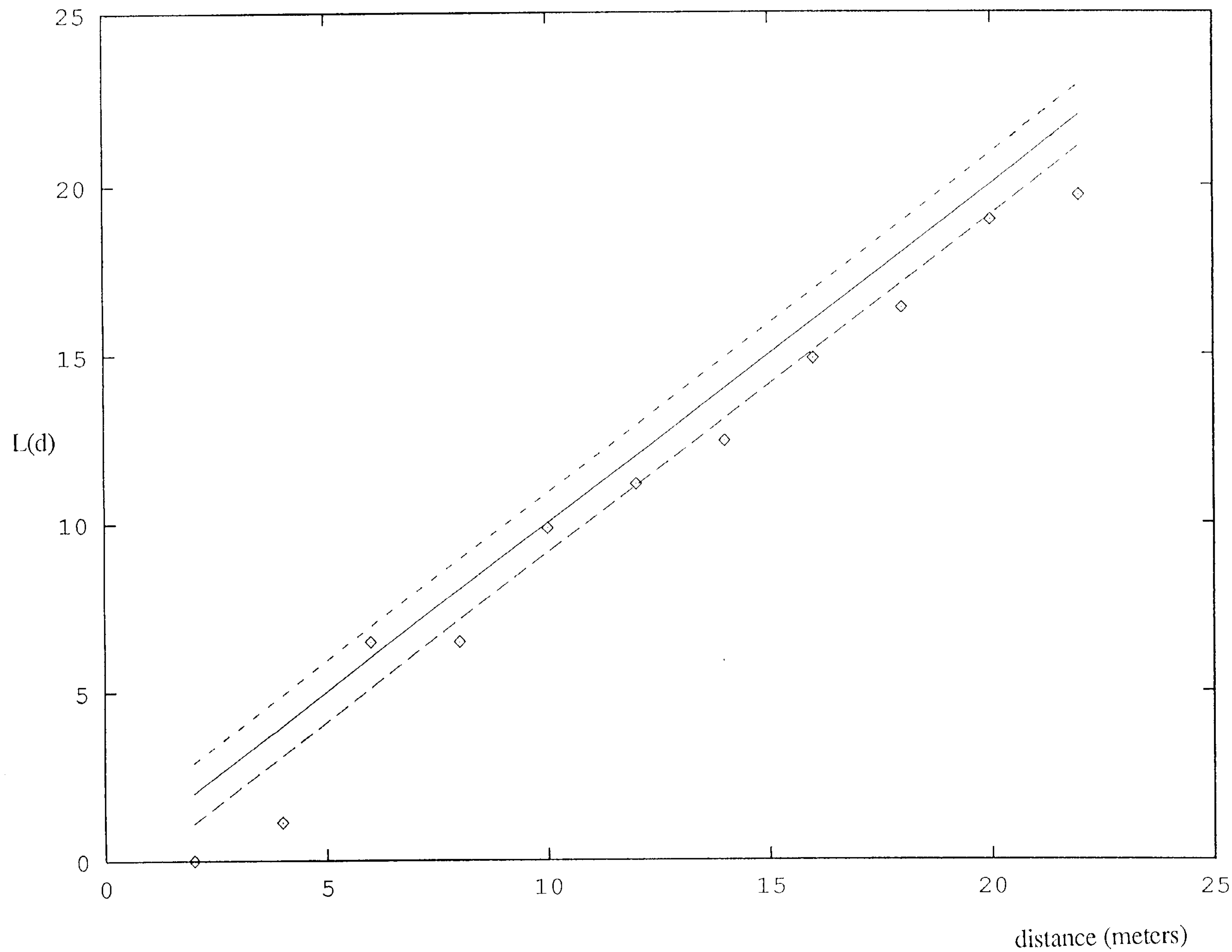


Figure 11

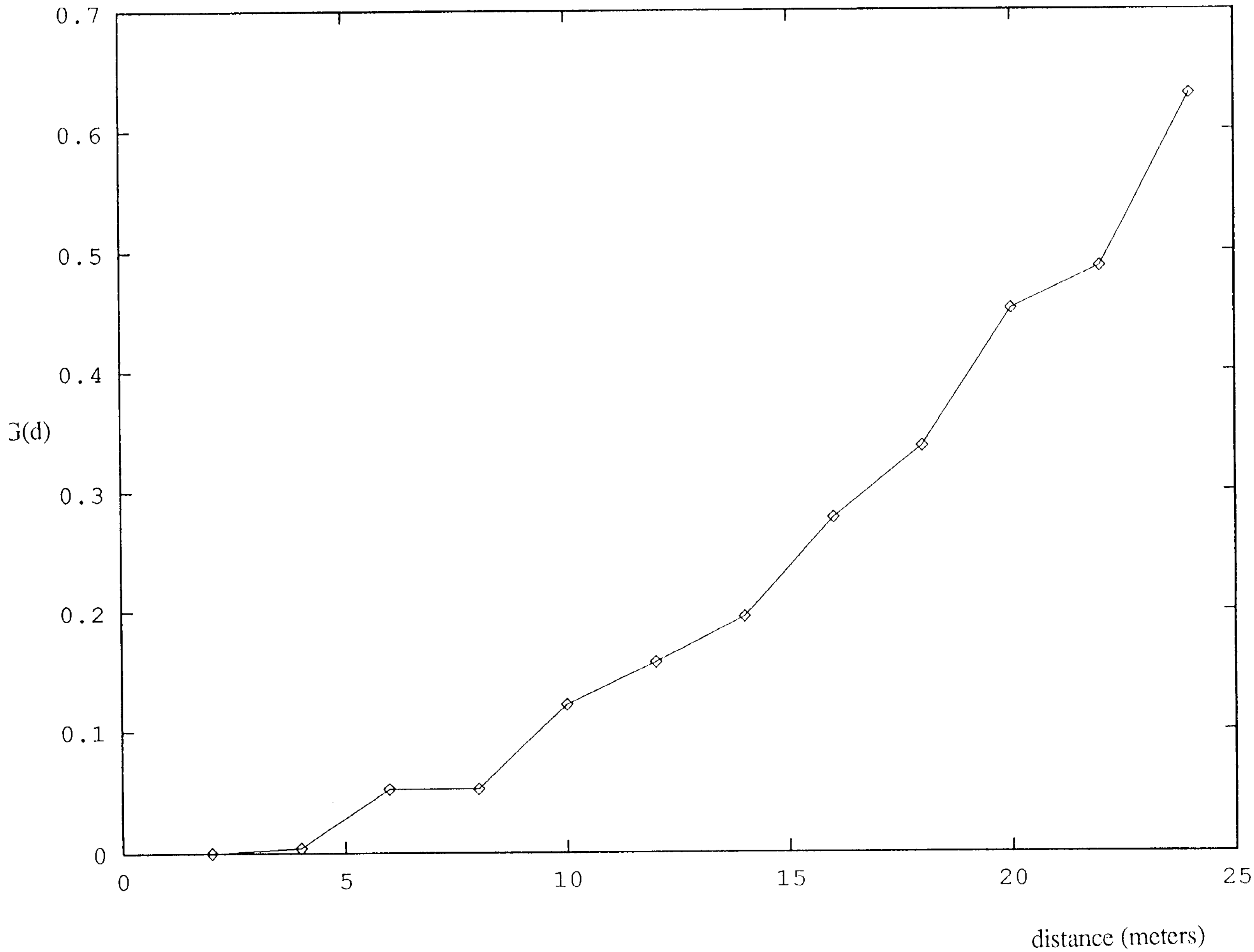


Figure 12

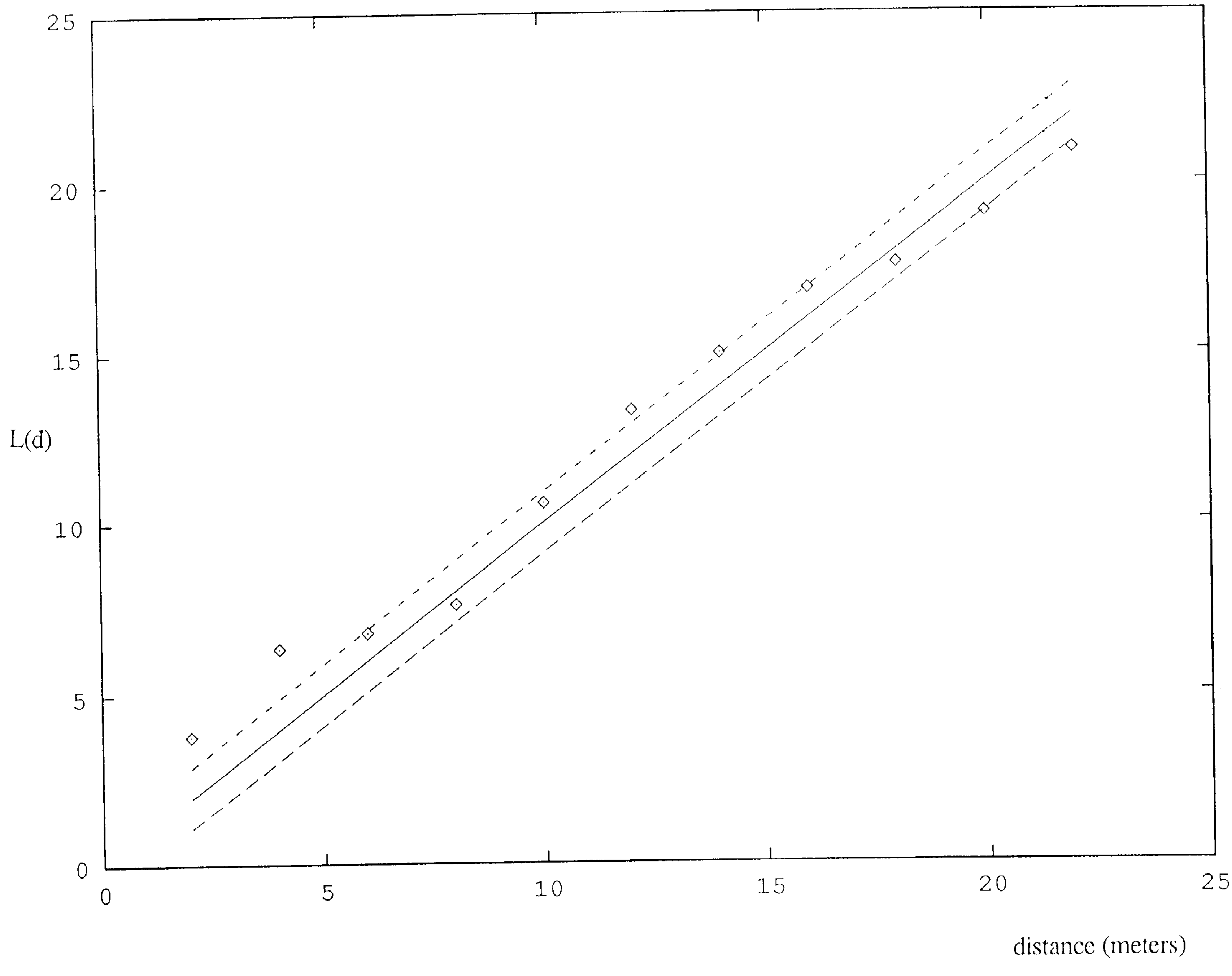


Figure 13

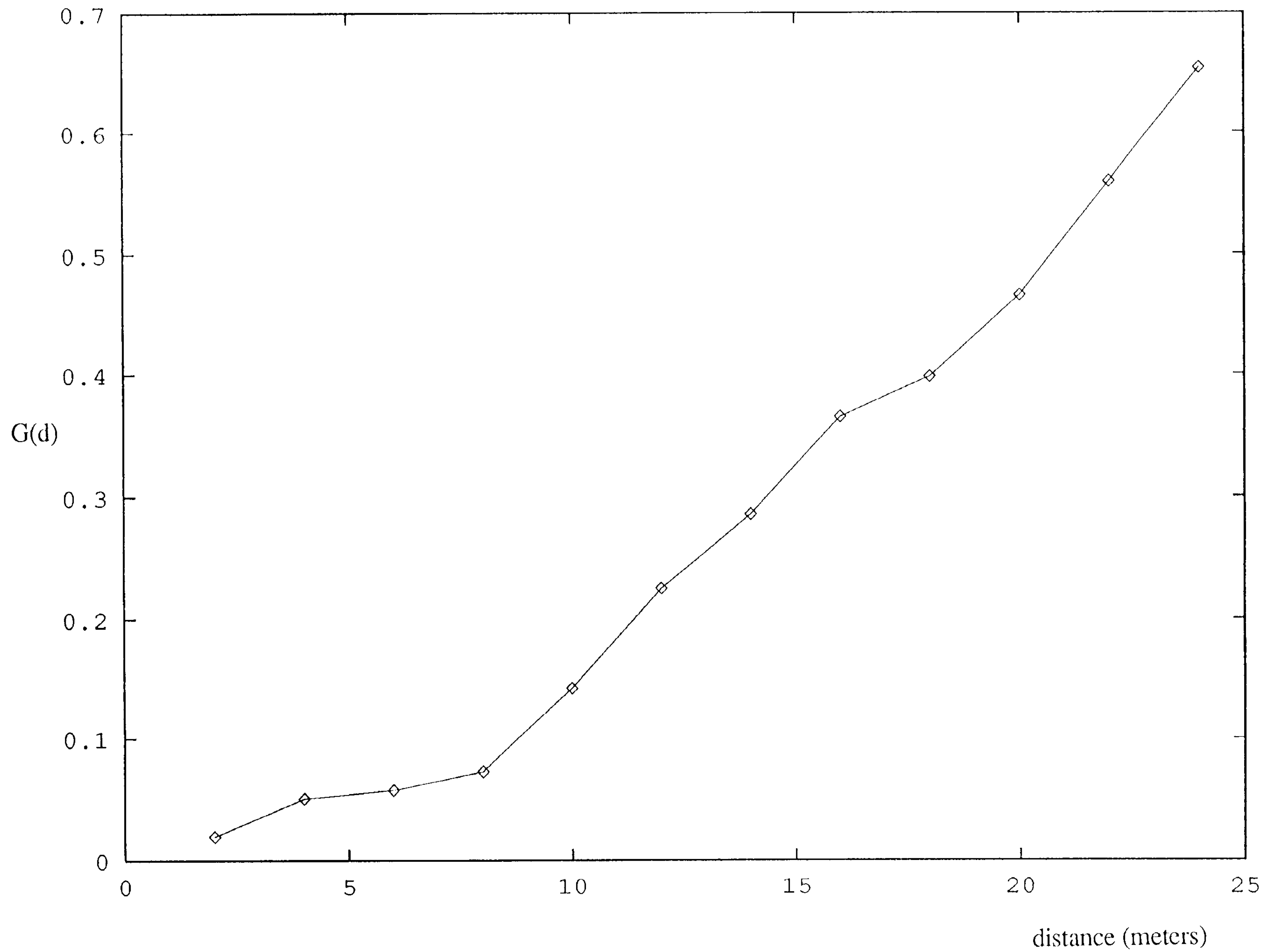
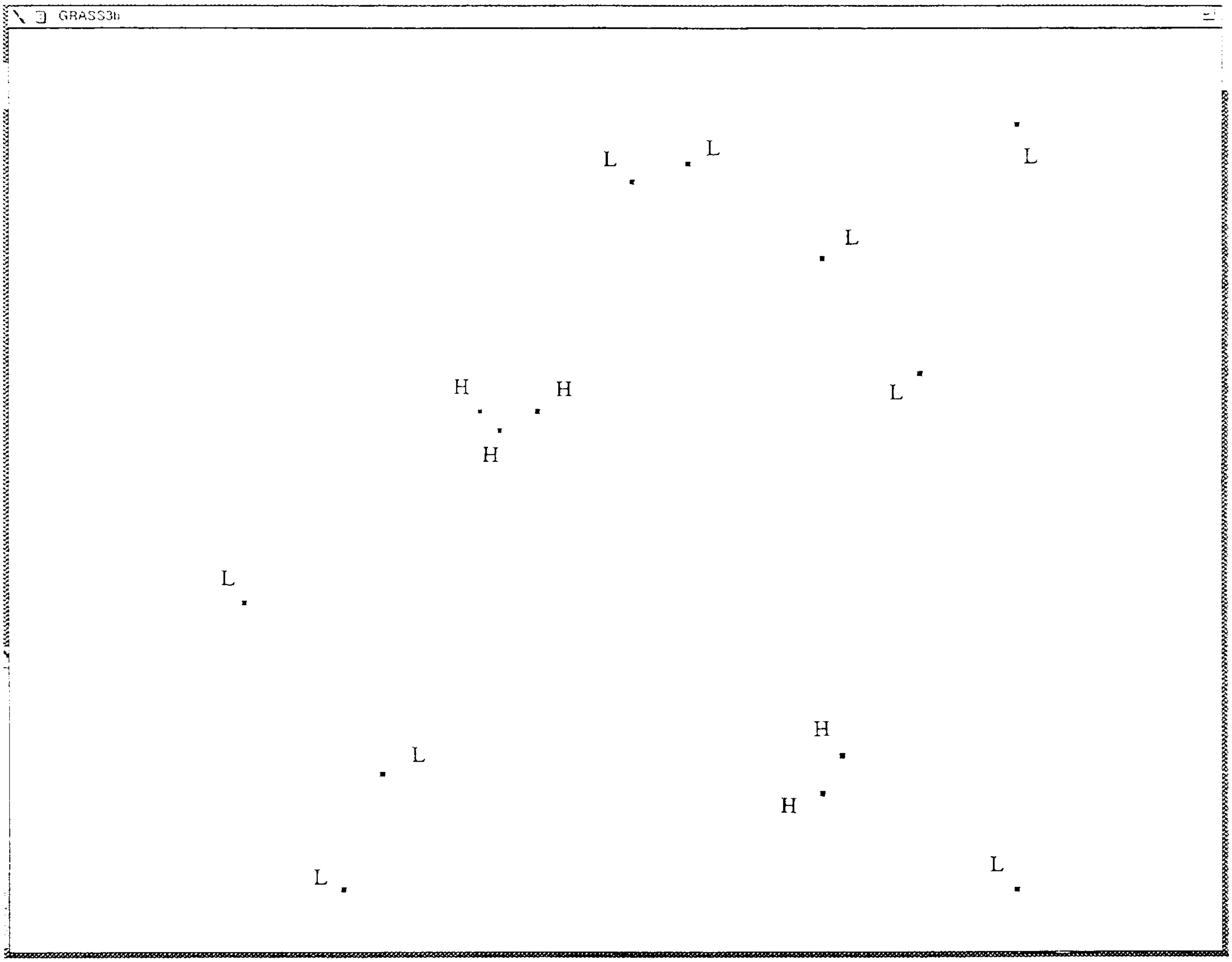


Figure 14



600067	4900080	#100
600070	4900080	#90
600068	4900079	#90
600085	4900060	#95
600086	4900062	#100
600095	4900095	#1
600055	4900070	#2
600060	4900055	#1
600090	4900082	#3
600095	4900055	#2
600062	4900061	#1
600075	4900092	#1
600085	4900088	#2
600070	4900051	#3
600078	4900093	#2

Figure 15

Gi(5.00) Statistic for point 0 at 600067(E) 4900080(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.458	0.143	0.031	1.793

Gi(5.00) Statistic for point 1 at 600070(E) 4900080(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.471	0.143	0.031	1.868

Gi(5.00) Statistic for point 2 at 600068(E) 4900079(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.471	0.143	0.031	1.868

Gi(5.00) Statistic for point 3 at 600085(E) 4900060(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.251	0.071	0.017	1.390

Gi(5.00) Statistic for point 4 at 600086(E) 4900062(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.242	0.071	0.017	1.318

Gi(5.00) Statistic for point 5 at 600095(E) 4900095(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 6 at 600055(E) 4900070(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 7 at 600060(E) 4900055(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 8 at 600090(E) 4900082(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 9 at 600095(E) 4900055(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 10 at 600062(E) 4900061(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 11 at 600075(E) 4900092(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.004	0.071	0.014	-0.575

Gi(5.00) Statistic for point 12 at 600085(E) 4900088(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 13 at 600070(E) 4900051(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.000	0.000	0.000	0.000

Gi(5.00) Statistic for point 14 at 600078(E) 4900093(N)

distance	Gi(d)	E(G)	V(G)	z-score
5.000	0.002	0.071	0.014	-0.591

G(d) Statistic for window 600050(E) 600100(E) 4900100(N) 4900050(N)
number of points: 15 area: 2500.000 sq.meters Case: i <> j

distance	G(d)	E(G)	V(G)	z-score
5.000	0.110	0.048	0.664	0.076

Figure 16

Gi(15.00) Statistic for point 0 at 600067(E) 4900080(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.461	0.214	0.043	1.194

Gi(15.00) Statistic for point 1 at 600070(E) 4900080(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.474	0.214	0.043	1.257

Gi(15.00) Statistic for point 2 at 600068(E) 4900079(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.474	0.214	0.043	1.257

Gi(15.00) Statistic for point 3 at 600085(E) 4900060(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.256	0.143	0.031	0.645

Gi(15.00) Statistic for point 4 at 600086(E) 4900062(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.247	0.143	0.031	0.591

Gi(15.00) Statistic for point 5 at 600095(E) 4900095(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.010	0.143	0.025	-0.832

Gi(15.00) Statistic for point 6 at 600055(E) 4900070(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.002	0.071	0.014	-0.591

Gi(15.00) Statistic for point 7 at 600060(E) 4900055(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.008	0.143	0.025	-0.845

Gi(15.00) Statistic for point 8 at 600090(E) 4900082(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.006	0.143	0.026	-0.854

Gi(15.00) Statistic for point 9 at 600095(E) 4900055(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.397	0.143	0.026	1.591

Gi(15.00) Statistic for point 10 at 600062(E) 4900061(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.012	0.214	0.035	-1.079

Gi(15.00) Statistic for point 11 at 600075(E) 4900092(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.577	0.357	0.048	1.003

Gi(15.00) Statistic for point 12 at 600085(E) 4900088(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.014	0.286	0.043	-1.312

Gi(15.00) Statistic for point 13 at 600070(E) 4900051(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.004	0.143	0.026	-0.867

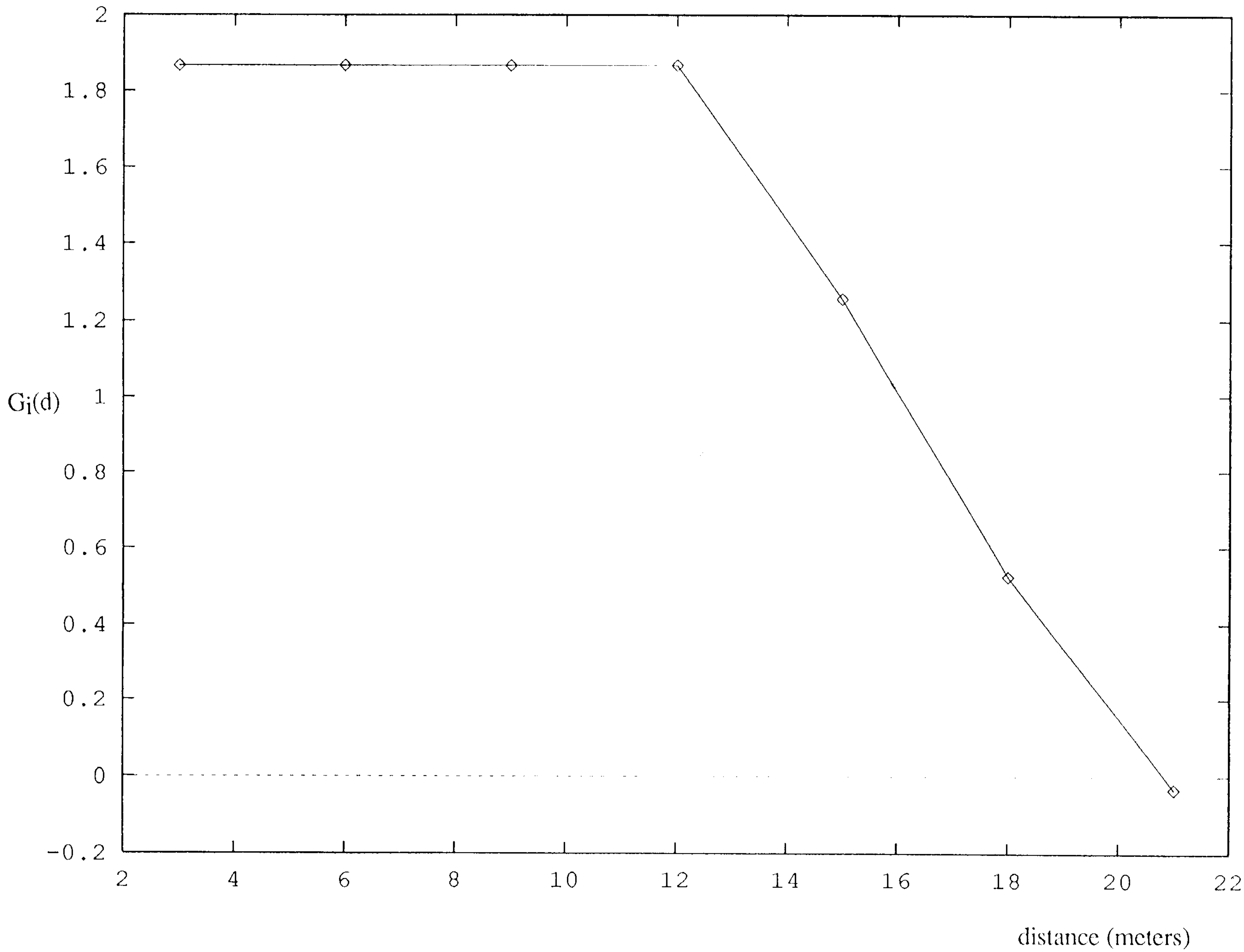
Gi(15.00) Statistic for point 14 at 600078(E) 4900093(N)

distance	Gi(d)	E(G)	V(G)	z-score
15.000	0.006	0.143	0.026	-0.856

G(d) Statistic for window 600050(E) 600100(E) 4900100(N) 4900050(N)
number of points: 15 area: 2500.000 sq.meters Case: i <> j

distance	G(d)	E(G)	V(G)	z-score
15.000	0.184	0.181	2.168	0.002

Figure 17



2
1.8
1.6
1.4
1.2
$G_i(d)$
1
0.8
0.6
0.4
0.2
0
-0.2
2
4
6
8
10
12
14
16
18
20
22
distance (meters)

Figure 18